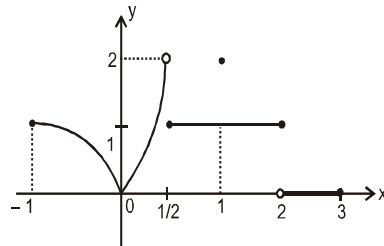


Topics : Fundamental of Mathematics, Function, Limits

Type of Questions		M.M., Min.
Single choice Objective (no negative marking) Q.1,2,3	(3 marks, 3 min.)	[9, 9]
Multiple choice objective (no negative marking) Q.4	(5 marks, 4 min.)	[5, 4]
Subjective Questions (no negative marking) Q.5,6,7	(4 marks, 5 min.)	[12, 15]
Match the Following (no negative marking) Q.8	(8 marks, 8 min.)	[8, 8]

- Total number of positive integers x for which $f(x) = x^3 - 8x^2 + 20x - 13$ is a prime number, is
(A) 1 (B) 2 (C) 3 (D) 4
- Let f be a real valued function such that for any real x
 $f(15 + x) = f(15 - x)$ and $f(30 + x) = -f(30 - x)$
Then which of the following statements is true ?
(A) f is odd and periodic (B) f is odd but not periodic
(C) f is even and periodic (D) f is even but not periodic
- Which of the following functions is **not** periodic, where $[\cdot]$ denotes greatest integer function
(A) $f(x) = 1^{|x|} + (-1)^{|x|}$ (B) $g(x) = 1^{[5x]} + (-1)^{[5x]}$
(C) $h(x) = 2^{|x|} - (-2)^{|x|}$ (D) $\phi(x) = 1^{|x|} - (-1)^{|x|}$
- Which of the following statements are true for the function f defined for $-1 \leq x \leq 3$ in the figure shown.



- $\lim_{x \rightarrow -1^+} f(x) = 1$
- $\lim_{x \rightarrow 2} f(x)$ does not exist
- $\lim_{x \rightarrow 1^-} f(x) = 1$
- $\lim_{x \rightarrow 0^+} f(x) = \lim_{x \rightarrow 0^-} f(x)$
- $\lim_{x \rightarrow c} f(x)$ exists at every c between -1 & 1
- $\lim_{x \rightarrow c} f(x)$ exists at every c between -1 & 0 .

5. Find the fundamental period of the functions

(i) $f(x) = \sin\left(2\pi x + \frac{\pi}{3}\right) + 2\sin\left(3\pi x + \frac{\pi}{4}\right) + 3\sin 5\pi x$

(ii) $f(x) = \sin\left(\frac{\pi}{3}x\right) + \cos\left(\frac{\pi}{4}x\right)$

6. If $f(x) = 4x^3 - x^2 - 2x + 1$ and $g(x) = \begin{cases} \text{Min}\{f(t) : 0 \leq t \leq x\} & ; 0 \leq x \leq 1 \\ 3 - x & ; 1 < x \leq 2 \end{cases}$ then find the value of

$g\left(\frac{1}{4}\right) + g\left(\frac{3}{4}\right) + g\left(\frac{5}{4}\right)$.

7. Identify the indeterminate forms (if any) in the following limits :

(i) $\lim_{x \rightarrow 0} \frac{\sin x^3}{x^2}$

(ii) $\lim_{x \rightarrow 0} \frac{\sin[x^2]}{[x^2]}$; [.] represents the greatest integer function

(iii) $\lim_{x \rightarrow 0} |x|^{\lfloor \sin^2 x \rfloor}$; [.] represents the greatest integer function

(iv) $\lim_{x \rightarrow 0^+} \frac{\operatorname{cosec}^{-1} x}{\cot^{-1} x}$

(v) $\lim_{x \rightarrow 0^-} \frac{\operatorname{cosec}^{-1} x}{\cot^{-1} x}$

8. Let $f(x) = x + \frac{1}{x}$ and $g(x) = \frac{x+1}{x+2}$.

Match the composite function given in Column-I with respective domains given in Column-II.

Column I

Column II

(A) $f \circ g(x)$

(p) $\mathbb{R} - \{-2, -5/3\}$

(B) $g \circ f(x)$

(q) $\mathbb{R} - \{-1, 0\}$

(C) $f \circ f(x)$

(r) $\mathbb{R} - \{0\}$

(D) $g \circ g(x)$

(s) $\mathbb{R} - \{-2, -1\}$

(t) $\mathbb{R} - \{-1\}$

Answers Key

1. (C) 2. (A) 3. (C) 4. (A B D)

5. (i) 2 (ii) 24 6. $\frac{5}{2}$

7. (i) $\frac{0}{0}$ (ii) not defined (iii) non indeterminate
(iv) not defined (v) not defined

8. (A) \rightarrow (s) ; (B) \rightarrow (q) ; (C) \rightarrow (r) ; (D) \rightarrow (p)

